QUESTION 2012

Group – A
(Multiple Choice Type Questions)

1. Choose the correct alternatives for any *ten* of the following:

i) The value of $\lim\limits_{x \to 0} \dfrac{x^2 - 4}{x - 2}$ is

   a) 1   b) 4   c) 0   ✓d) none of these

ii) If $\phi(x, y) = 0$ then $\dfrac{dy}{dx} =$

   a) $\dfrac{\phi_x}{\phi_y}$ b) $\dfrac{\phi_y}{\phi_x}$ c) ϕ_y ✓d) none of these

iii) The value of y_n, when $y = e^{-x}$ is

   a) e^{-x} b) $(-1)^n$ ✓c) $(-1)^n e^{-x}$ d) none of these

iv) If $A = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}$, $A^2 =$

   a) A ✓b) $3A$ c) unit matrix   d) $2A$

v) The diagonal of real skew-symmetric matrix is

   a) 1   b) -1 ✓c) 0   d) 2

vi) If in a group $(G, o), x \, o \, x = x$, then

   a) $x =$ null element   b) this relation is not valid

   ✓c) $x = e$ d) $x \neq x^{-1}$

vii) The value of $\int (\cos^2 x - \sin^2 x)\, dx$ is

   a) $\sec x \tan x$ b) $\sin 2x$ ✓c) $\dfrac{1}{2}\sin 2x$ d) $-\dfrac{1}{2}\sin 2x$

viii) The polar equation $r=4\sin\theta$ represents a

a) circle   b) ellipse   ✓c) straight line   d) none of these

ix) If a,b and c are roots of $x^3-3x+9=0$ then $a^2+b^2+c^2$ is

✓a) 6   b) -6 c) 0   d) 1

x) Solution of the equation $x^3+2x+3=0$ will give

✓a) no real positive roots but one real negative root

b) two real positive roots and one real negative root

c) one real positive root and two imaginary roots

d) two real negative roots and only one imaginary root

xi) $\int_{-1}^{2}|x|\,dx$ is

a) 3   b) 5   ✓c) 5/2   d) 3/2

xii) Which of the following does not satisfy Rolles theorem in $[-2, 2]$?

a) x ✓b) $1/x$ c) $1/(x-5)$ d) x^2-5

Group – B
(Short Answer Type Questions)

2. Out of 440 students, 112 students read German, 120 students read French and 168 read Spanish. Of these 32 read French and Spanish, 40 read German and Spanish, 20 read German and French, while 12 read all the three subjects. How many students

a) do not read any of the three languages

b) read just one language?

See Topic: SET THEORY, Short Answer Type Question No. 6.

3. Evaluate $\int_{0}^{\pi}\log\tan x\,dx$.

See Topic: DEFINITE INTEGRALS, Short Answer Type Question No. 11.

4) If $f(x,)=\begin{cases}\dfrac{x^2-xy}{x+y},(x,y)\neq(0,0),\\ 0,(x,y)=(0,0)\end{cases}$ What is the value of $f_x(0,0), f_y(0,0)$.

See Topic: FUNCTIONS OF SEVERAL VARIABLES, Short Answer Type Question No. 5.

5. Obtain a relation between p, q and r so that $x^3 + px^2 + qx + r = 0$ has 3 roots that are in A.P.

See Topic: POLYNOMIAL, Short Answer Type Question No. 18.

6. Evaluates $\lim\limits_{x \to 0} \dfrac{\operatorname{cosec} x - \cot x}{x}$.

See Topic: LIMIT, CONTINUITY & DIFFERENTIABILITY, Short Answer Type Question No. 12.

Group – C
(Long Answer Type Questions)

7. a) Show that $\{1, \omega, \omega^2\}$, where $\omega^3 = 1$ forms a commutative group in respect of multiplication.

b) If $y = \sin^{-1} x$, then prove that $(1 - x^2) y_{n+2} - (2n+1) x y_{n+1} - n^2 y_n = 0$

c) In the mean value theorem $f(x+h) = f(x) + h f'(x + \theta h)$, if $f(x) = px^2 + qx + r \, (p \neq 0)$, then show that $\theta = \dfrac{1}{2}$.

a) See Topic: BINARY COMPOSITION, Short Answer Type Question No. 7.
b) See Topic: SUCCESSIVE DIFFERENTIATION, Short Answer Type Question No. 10.
c) See Topic: EXPANSION OF FUNCTIONS, Short Answer Type Question No. 6.

8. a) Reduce the equation $3x^2 + 2xy + 3y^2 - 16x + 20 = 0$ into canonical form and hence determine the nature of the conic.

b) Find the nature of the conic $\dfrac{8}{r} = 4 - 5\cos\theta$.

c) Expand e^x in ascending powers of x by Taylor's series.

a) See Topic: GENERAL EQUATION OF SECOND DEGREE, Short Answer Type Question No. 2.
b) See Topic: POLAR EQUATIONS, Short Answer Type Question No. 1.
c) See Topic: EXPANSION OF FUNCTIONS, Short Answer Type Question No. 7.

9. a) Solve using Carden's method: $x^3 - 9x + 28 = 0$.

b) If by a transformation of motion of co-ordinate axes, the expression $ax^2 + 2hxy + by^2$ changes into $a'x'^2 + 2h'x'y' + b'y'^2$, then show that $ab - h^2 = a'b' - h'^2$.

a) See Topic: POLYNOMIAL, Short Answer Type Question No. 6.
b) See Topic: TRANSFORMATION OF CO-ORDINATES, Long Answer Type Question No. 7.

10. a) Solve the equations by matrix inversion method:

$$x + y + z = 4$$
$$2x - y + 3z = 1$$
$$3x + 2y - z = 1$$

b) If $u = \tan^{-1} \dfrac{x^2 + y^2}{x + y}$, show that $x\dfrac{\partial u}{\partial x} + y\dfrac{\partial u}{\partial y} = \dfrac{1}{2}\sin 2u$.

c) Evaluate: $\displaystyle\int \dfrac{x-1}{(x-2)(x-3)} dx$.

a) See Topic: MATRICES, Long Answer Type Question No. 6.

b) See Topic: FUNCTIONS OF SEVERAL VARIABLES, Short Answer Type Question No. 6.

c) See Topic: INDEFINITE INTEGRATIONS, Short Answer Type Question No. 11.

11. a) Give the definition of a ring with two binary composition. Let H be the set of all matrices $\left\{ \begin{pmatrix} a & b \\ c & d \end{pmatrix} : ad - bc = 1 \right\}$. Prove that H forms a non-commutative group.

b) Apply Descerte's rule of sign to find the nature of the roots of the given equation:
$x^4 + qx^2 + rx - s = 0$ (where q, r, s being positive).

c) Evaluate: $\displaystyle\lim_{n \to \infty}\left[\dfrac{1}{n^2 + 1^2} + \dfrac{1}{n^2 + 2^2} + \dots\dots + \dfrac{1}{n^2 + n^2} \right]$.

a) See Topic: BINARY COMPOSITION, Short Answer Type Question No. 8.

b) See Topic: POLYNOMIAL, Short Answer Type Question No. 19.

c) See Topic: LIMIT, CONTINUITY & DIFFERENTIABILITY, Short Answer Type Question No. 12.